Towards Optimization of Reversible Circuits

LogDynamics
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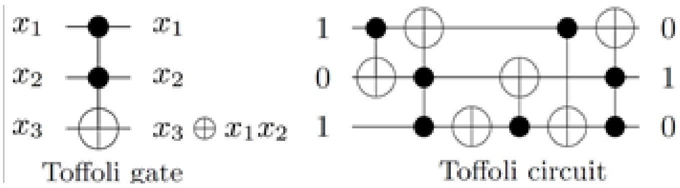
Reversible logic circuit draws motivation from Landauer's principle which pointed out that traditional logic dissipates a certain amount of power for every bit of information loss. Later, other researchers showed that this power dissipation can be eliminated if the computation becomes reversible. Therefore, reversible circuits can be a suitable choice in the design of low-power digital circuits. Besides that, such circuits find major application in quantum computing, DNA computing, nano-scale logic design, etc. In future this enabling technology will be applied in the areas like logistics.

Unlike traditional logic circuits, a reversible circuit has an equal number of inputs and outputs, and maps each input patterns to unique output patterns. Usually, this kind of circuit is composed of reversible gates such as Toffoli gate and the Fredkin gate. A reversible gate consists of a set of control lines and a target line. The target line is inverted if all the control lines are set to logic 1. Otherwise, the target line remains un-changed. The figure below shows the Toffoli gate and a reversible circuit. The solid black circles indicate the control connections and the symbol denotes the target line.
The complexity of the reversible circuit is measured by means of quantum cost. The quantum cost depends on the number of control lines of a reversible gate. As an example, quantum costs for a selection of the Toffoli gate configurations are given in the table. Normally, the quantum cost of a reversible circuit is calculated as the sum of quantum cost of an individual gate of that circuit.



Toffoli gate with 2 control lines and a Toffoli circuit

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Number of Control lines	Quantum cost of Toffoli gate		Number of Control lines	Quantum cost of Toffoli gate
0	1		6	44
1	1		7	64
2	5		8	76
3	14		9	96
4	20		10	108
5	32		11	132

Quantum cost for Toffoli gates

Research Trends in Reversible Logic

Previous studies indicate that synthesis is the most important step in the design of reversible circuits. This is a process of generating a compact reversible circuit from a given specification. To date, researchers have developed several synthesis methods for designing this kind of circuit. In particular, synthesis has been intensely studied based on various specifications, e.g. truth-table, binary decision diagram, exclusive sum-of-products (ESOPs). But often, these approaches do not generate circuits with minimal quantum cost. Therefore, post-synthesis optimization is applied to reduce the cost of the circuit.

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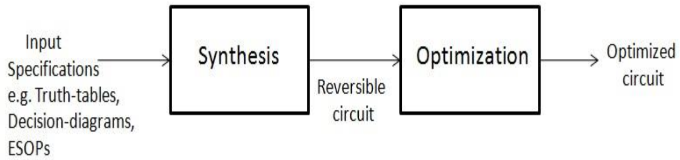

A simple design flow of reversible circuits

In the literature, optimization approaches considering template matching or common control reductions have been established. In template matching, a search method is used to look for a sub-circuit from a given circuit which can be replaced by an equivalent circuit of low quantum cost. Although, this approach produces a better result in terms of quantum cost, but significantly increases the run-time with the number of applied templates and the number of gates in the circuit. Hence, this is not suitable for larger circuits. In contrast, an alternative approach focuses on the common control connections of a given circuit and reduces them by adding a line to that circuit. This results in significant reduction in the overall quantum cost of the circuit. However, this is achieved by adding extra lines to the circuit, thereby, increasing the number of circuit lines. This is a major drawback, since the number of circuit lines corresponds to the number of quantum bits (qubits) – which are considered as a limited resource.

Research Objective

As discussed above, these optimization methods have their own drawbacks, which restrict their applicability to a certain extent. Therefore, it is crucial to establish an optimization technique which is more efficient compared to the existing methods. In this regard, the following problem can be formulated:

For a given reversible circuit, how an optimization method reduces the quantum cost without any additional overhead such as runtime and circuit lines.

To solve the problem, a new optimization technique needs to be investigated; the main focus of the research work is to develop an efficient optimization technique for reversible circuits. Here, „efficient“ is defined by minimal quantum cost of resulting optimized circuits with no additional circuit lines and smaller runtime.

The common control connections can be a major focus during optimization. This is caused by the fact that gates with many control lines are a main reason for high quantum costs in the respective reversible circuits, thereby, reducing their amount allows substantial reductions. This was earlier exploited with an additional line. This idea can be explored without any additional lines. One possible solution is to identify cascades of gates with common control lines and different target lines and replace them with functionally equivalent gate cascades of lower quantum cost. Further study on this is still required.

Conclusion

The purpose of the research work is to develop a suitable optimization method for reversible circuits. To this end, a very brief idea is mentioned, based on which, a possible optimization method can be designed. Future work involves the detailed inves-tigation of the proposed idea and experimental confirmation of its usefulness.